Analysis of the data plotted in Figs. 2 and 3 shows that the magnitude and nature of change of the deflection of the plate depends on the shape of the wave formation, the position of the loading front, and its velocity. With increase of velocity v, the deflections decrease.

For $t_0 > t_*$, assuming $Q_{mn} = 4$, $m = 1, 3, 5, \ldots$, $n = 1, 3, 5, \ldots$, $Q_{kj} = 16$, $k = 1, 3, 5, \ldots$ and $j = 1, 3, 5, \ldots$, and using the values obtained for w_{mn} and w'_{mn} as the starting values, the further change of deflection can be determined.

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FRACTURE OF CYLINDRICAL SHELLS BY THE ACTION OF PERIODIC SHOCK WAVES

M. A. Il'gamov and A. V. Sadykov

UDC 539.37

It is well-known that in a closed tube, to one end of which is applied a sinusoidal piston movement, nonlinear longitudinal oscillations originate, which in the vicinity of the natural frequencies transform to periodic shock waves [1-13]. Similar oscillations originate during the unstable operation of the combustion chamber of engines [14-16]. In the experiments carried out up to now, the amplitude achieved 0.36 bar with an average pressure in the tube of 1 bar [4, 8, 13]. Forced axisymmetrical oscillations of thin-walled shells under the action of periodic shock waves inside their cavity have been studied in [17]. A relatively good carrying capacity is characteristic of them. This is explained by the fact that the oscillations are accompanied by a predominantly stretching–compression of the cross-section of the shell. Moreover, experiments were carried out at frequencies close to the natural frequencies of the gas column $\omega_k = k\pi a/L$ (in order to produce shock waves in the gas) and axisymmetrical oscillations of the shell $\Omega_i \approx \Omega_0 \approx c/R$ remote from the natural frequencies. Here a and c are the propagation velocities of sound in the gas and in the shell; L and R are the total length of the tube and the radius of the middle surface of the shell.

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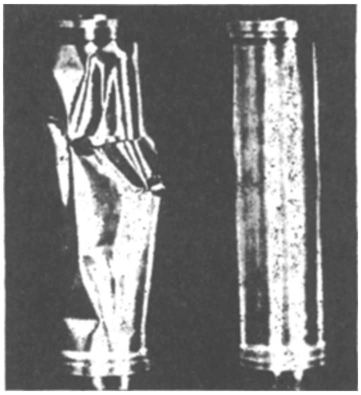

Fig. 1

For shells with the most usual dimensions, the latter condition is always fulfilled. At present, there are no piston machines with sufficiently high revolutions to be able to excite axisymmetrical oscillations of the shell predominantly of the radial type by the method being considered here.

In the absence of axisymmetrical oscillations, the shell, in the case of oscillations with wave formation around the periphery, is found to be extremely vulnerable to longitudinal waves of the gas in its cavity. As a result of wave formation, fatigue cracks of large amplitude form on the surface of the shell in all for 20-25 cycles after the start of the bending oscillations (after knocking). After several cycles, the shell is completely crushed (Fig. 1).

An experimental investigation of this phenomenon was carried out on an installation [17] which consists of an automobile engine with the head removed, a dc electric motor for turning the crankshaft of the engine, belt drive, and pulleys. The main tube, with a wall thickness of 3 mm and internal diameter 2R = 82 mm, was attached to a cylinder of the engine. To its other end, the thin-walled shell being investigated was joined by means of a special coupling. To the end of the latter was clamped a thick-walled end-plate. The total length of the air column was 3530 mm. The air was at room temperature before the start of the experiment and the pressure was 1 atm. The oscillations in the air column are created by the movement of the engine piston, with a stroke of 110 mm.

Particular attention was paid to the accuracy of manufacture of the shells to be tested. They were turned out of Duralumin D16 on a screw-cutting lathe. For this, a specially made mandrel was used for reducing deformation during turning. The surfaces of the shells after the lathe treatment were subjected to fine polishing. The total deviation of the dimensions due to ovalicity, conicity, and eccentricity did not exceed 0.015 mm, which amounts to less than 10% of the wall thickness. Monitoring was carried out by means of dial gauges with 0.005 mm gradations. A number of shells were prepared, with a wall thickness of 0.19 to 0.25 mm.

Measurement of the pressure oscillations in the gas was effected by means of piezoelectric sensors, installed at several places along the length of the thin-walled tube. Some of the sensors were located close to the joint with the shell under test. Signals from the sensors, after the U4-1 amplifier (through its low-impedance output), were applied to an N-102 loop oscillograph. The frequency of the oscillations was determined by means of an electronic frequency meter ChZ-24 with gradations of 1 Hz and controlled in parallel by an F 5080 frequency meter with gradations of 0.1 Hz. The tenso-sensors for recording the deformations of the shell after the start of the bending oscillations were put out of service immediately, in view of the high levels of the deformations. The forms of the oscillations of the shell were determined by an SKS-1 moving-picture camera (speeds of 170 to 650 frames/sec were used). The shell was illuminated by two four-lamp illuminators.

A smooth increase of voltage in the electric motor led to a smooth increase of the engine revolutions and the frequency of excitation. The record of the readings of the frequency meter and of the pressure sensors and startup of the high-speed moving-picture camera were effected synchronously. Up to an excitation frequency of $0.9\ \omega_1$, where ω_1 is the lowest natural frequency of the column of air at which a kink occurs on the curve in the pressure diagram, the combined axisymmetrical oscillations of the gas and of the shell are described well by linear theory. With the appearance of the kink and discontinuities, the pressure can be calculated only on the basis of the nonlinear equations of hydromechanics. However, the oscillations of the shell still are axisymmetrical, remote from resonance oscillations, and can be described by linear equations. In this case, the interaction between the air and the shell is weak, so that the readings of the pressure sensors differ but little from

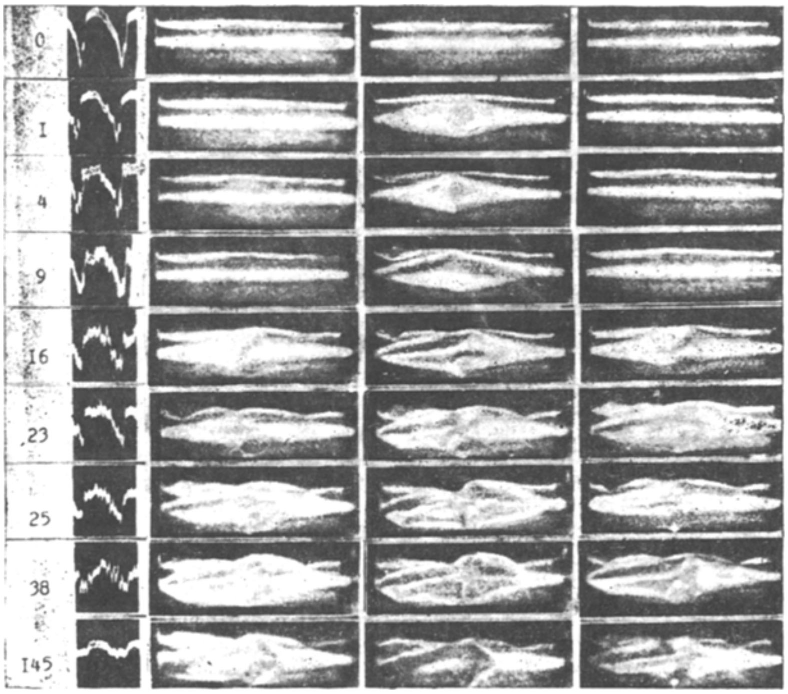

Fig. 2

the readings in the case when the shell being tested is replaced by a thick-walled tube. Finally, the instant arrives when the shell–gas system loses stability of the axisymmetrical forms of the oscillations and several waves appear round the periphery.

Figure 2 shows the pressure sensor readings (left-hand column) and the moving-picture frames of three positions of the shell for a cycle of oscillations (right-hand columns). From 4 to 15 photographs were produced for each cycle of oscillations; here only photographs are shown which correspond to the instant of maximum deflection inside (center column) and the instants when the shell is tending to assume a circular form. The last cycle of oscillations of the system with axisymmetrical shape is denoted by 0, after which follows cycle 1, when knocking of the shell occurred. In all the shells tested, four waves appeared around the periphery and one half-wave along the length. The pattern up to cycle 9 remains approximately identical, and only the amplitud is increased. From the instant of knocking a high-frequency component appears in the pressure diagram, and the amplitude has a tendency to some reduction. Later (cycles 16 and 23), a sharp folding takes place, which no longer allows the shell to assume a circular shape at specified instants. In cycle 25, the appearance of cracks leads to a reduction of the pressure amplitude. Subsequently, the pressure in the tube falls sharply (cycles 38 and 145). We note that the maximum kink in the pressure curve reached 0.72 bar on the equipment.

These results refer to a shell with an average thickness of 0.23 mm. The loss of its circular shape occurs at an excitation frequency of $f \approx 42.5$ Hz. In the next 25-30 cycles it can be assumed to be unchanged. With reduction of the thickness of the shell, distortion occurs even more rapidly. Figure 3 shows moving-picture frames for a shell with thickness 0.21 mm, where the wave formation has an even more vigorous nature. Here permanent dents appeared on the sixth cycle and on cycle 25 there are already strongly developed cracks. Knocking appeared at about this excitation frequency, and also above.

Shells of Duralumin D16 with a wall thickness of 0.28 and greater did not lose stability of their circular shape during numerous tests.

It should be mentioned that there are considerable difficulties in carrying out the tests described. In contrast from the investigation of the axisymmetrical form of oscillations of the system, when the working conditions are within the scope of the principles of safety techniques, from the instant of the first knocking of the shell, noise of an exceptionally high level appears. Not only the equipment is subjected to intense vibration, but also the walls of the main three-storied building, and it appears in the semibasement in the foundations, insulated with concrete. All this makes it difficult to carry out the necessary measurements.

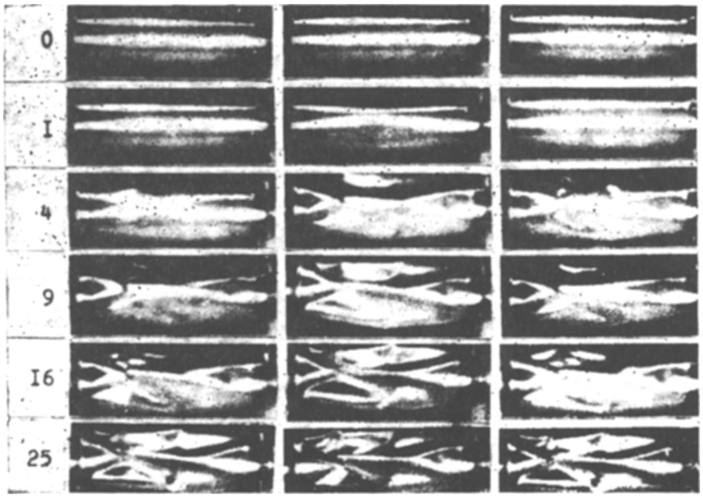

Fig. 3

At present, there is no theory for the interaction between a shell and a compressed liquid in the formulation considered here. Individual attempts (for example, in [18]) cannot claim to describe this experiment, characterized by strong nonlinearities in the shell, in the gas, and at their moving surface of contact. It can only be said that the number of waves around the periphery, corresponding to the minimum intrinsic significance of the problem concerning the stability of a cylindrical shell under the action of a static pressure, is close to the number of waves which was obtained in the experiment. For shell material with modulus of elasticity $E = 7.06 \cdot 10^{10}\ N/m^2$, Poisson coefficient $\nu = 0.3$, density $\rho = 2.77\ kg/m^3$, and dimensions $R = 41$ mm, $l = 222$ mm, and $h = 0.23$ mm, according to the formula in [18]

$$n = \sqrt[4]{6\pi^2\sqrt{1-\nu^2}}\,\sqrt{R/l}\,\sqrt[4]{R/l} = 4.38.$$

With a shell thickness of 0.21 mm, the number $n = 4.48$. Obviously, the effect of air on the number of waves n is small.

The natural oscillation frequency of the shell, corresponding to a form with a single half-wave along the generatrix and with four waves around the periphery, is equal to 218 Hz (for a thickness of 0.23 mm). Its ratio to the frequency of excitation and, consequently, to the transmission frequency of the shock waves at which dynamic loss of stability occurred (42.5 Hz) amounts to 5. This ratio is maintained also for shells with smaller thicknesses. It is probable that the frequency of the nonlinear oscillations followed after each cycle by knocking, zones of depression, and exhaust is considerably less than the frequency of the linear oscillations. In the experiment it was found to be equal to the excitation frequency (i.e., 42.5 Hz).

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STRESS CONCENTRATION NEAR AN INHOMOGENEITY AND EXPERIMENTAL CLARIFICATION OF THE COUPLE-STRESS EFFECT

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UDC 539.30 + 620.171.5

Following the fundamental papers [1-3], many authors have recently exerted considerable efforts to develop a new mechanics of the microcontinuum in which displacements of microelements are taken into account. To apply these theories it is required to determine a sufficiently large quantity of new elastic constants. The effects which appear because of taking account of the microelement displacements have been examined theoretically by a number of authors. However, there is quite little experimental work in which the effect of this accounting would be explained and new elastic constants would be determined in materials [4-6]. The purpose of the present paper is to clarify the effect of the influence of couple stresses on stress concentration near an inhomogeneity in the case of plane strain by the experimental method of photoelasticity.

Stress concentration near a circular cylinder of radius a exactly coincident with a cavity and imbedded in an infinite medium (strip), which is subject to the action of a uniform load p at infinity, is considered. The circular cylinder (inhomogeneity) and the external medium have different elastic constants: the shear modulus and Poisson ratio G_1, ν_1 and G_2, ν_2; l_1 and l_2 (new elastic constants introduced by the couple-stress theory of elasticity), respectively.

Let us take an r, θ polar coordinate system. Let σ_r, σ_θ, $\tau_{r\theta}$, $\tau_{\theta r}$ denote the stress tensor components and μ_r, μ_θ, the couple-stress tensor components.

Taking the stress function in the form from [2] and forming the boundary conditions of complete contact on the contour of the inhomogeneity (at $r = a$), we find the solution for stresses according to the couple-stress theory of elasticity [1, 2], which is obtained in closed form in terms of the modified Bessel functions I and K. According to the classical theory in which couple-stresses are neglected, the solution is obtained by a passage to the limit when $l_1 = l_2 = 0$.

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